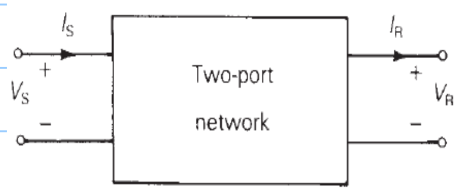
Ch 5. Transmittion Lines: Steady-State Operation

Note Title 3/24/2014

5.1. Medium and Short Line Approximation:

ABCD parameters:

$$\left.\begin{aligned} V_S &= AV_R + BI_R \quad \text{volts} \\ I_S &= CV_R + DI_R \quad \text{A} \end{aligned}\right\} \quad \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}, \qquad AD - BC = 1$$



a) Short transmittion line: ($\leqslant 80$ km)

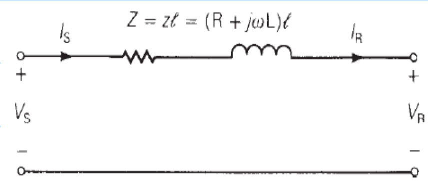
$z = \mathrm{R} + j\omega\mathrm{L}$ Ω/m, series impedance per unit length

$y = \mathrm{G} + j\omega\mathrm{C}$ S/m, shunt admittance per unit length

$Z = zl$ Ω, total series impedance

$Y = yl$ S, total shunt admittance

l = line length m



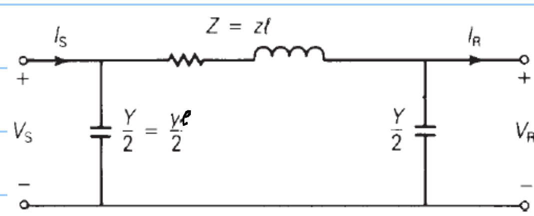
By KVL & KCL

$$\left.\begin{aligned} V_S &= V_R + ZI_R \\ I_S &= I_R \end{aligned}\right\} \quad \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} 1 & Z \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

b) Medium-lenght transmittion line: (80 km ~ 250 km)

By KVL:

$$\boxed{\begin{aligned} V_S &= V_R + Z\left(I_R + \frac{V_R Y}{2}\right) \\ &= \left(1 + \frac{YZ}{2}\right) V_R + ZI_R \end{aligned}}$$



(nominal π circuit)

By KCL:

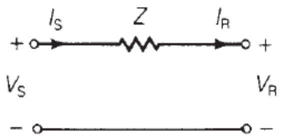
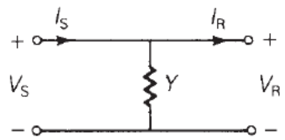
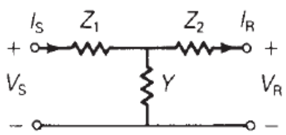
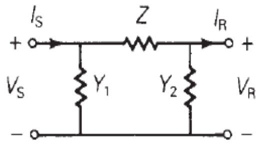
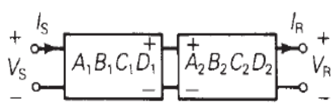
$$I_S = I_R + \frac{V_R Y}{2} + \frac{V_S Y}{2}$$

$$I_S = I_R + \frac{V_R Y}{2} + \left[\left(1 + \frac{YZ}{2}\right) V_R + ZI_R\right] \frac{Y}{2}$$

$$= Y\left(1 + \frac{YZ}{4}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} \left(1+\frac{YZ}{2}\right) & Z \\ \hline Y\left(1+\frac{YZ}{4}\right) & \left(1+\frac{YZ}{2}\right) \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

For short line: series impedance and shunt admittance. For medium length line: T circuit and Π circuit.

Circuit	*ABCD* Matrix
 Series impedance	$\left[\begin{array}{c\|c} 1 & Z \\ \hline 0 & 1 \end{array}\right]$
 Shunt admittance	$\left[\begin{array}{c\|c} 1 & 0 \\ \hline Y & 1 \end{array}\right]$
 T circuit	$\left[\begin{array}{c\|c} (1+YZ_1) & (Z_1+Z_2+YZ_1Z_2) \\ \hline Y & (1+YZ_2) \end{array}\right]$
 Π circuit	$\left[\begin{array}{c\|c} (1+Y_2Z) & Z \\ \hline (Y_1+Y_2+Y_1Y_2Z) & (1+Y_1Z) \end{array}\right]$
 Series networks	$\left[\begin{array}{c\|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]\left[\begin{array}{c\|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right] = \left[\begin{array}{c\|c} (A_1A_2+B_1C_2) & (A_1B_2+B_1D_2) \\ \hline (C_1A_2+D_1C_2) & (C_1B_2+D_1D_2) \end{array}\right]$

Voltage Regulation:

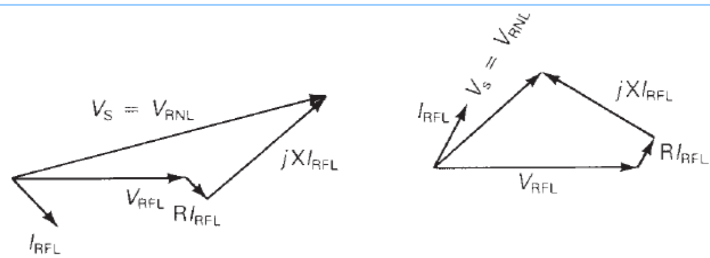
V_{RNL} : No-load voltage at receiving-end

V_{RFL} : Full load voltage at receiving-end

$$\text{percent VR} = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} \times 100$$

FIGURE 5.5

Phasor diagrams for a short transmission line



(a) Lagging p.f. load (b) Leading p.f. load

$V_{RNL} = \frac{V_S}{A}$, $I_{RNL} = 0$

percent VR should be ±5% (10%)

* Transmittion line considerations:

$\leq 80km$ ① Short line : % VR + the thermal limit.(current-carrying capacity)

80km ~ 300km ② Medium-lenght line : % VR + the thermal limit + the voltage drop. ($V_R/V_S \geq 0.95$)

$\geq 300km$ ③ Longer line : % VR + the thermal limit + the voltage drop + S.S stability limit.

EXAMPLE 5.1 ***ABCD* parameters and the nominal π circuit: medium-length line**

A three-phase, 60-Hz, completely transposed 345-kV, 200-km line has two 795,000-cmil 26/2 ACSR conductors per bundle and the following positive-sequence line constants:

$$z = 0.032 + j0.35 \quad \Omega/\text{km}$$

$$y = j4.2 \times 10^{-6} \quad \text{S/km}$$

Full load at the receiving end of the line is 700 MW at 0.99 p.f. leading and at 95% of rated voltage. Assuming a medium-length line, determine the following:

a. *ABCD* parameters of the nominal π circuit

b. Sending-end voltage V_S, current I_S, and real power P_S

c. Percent voltage regulation

d. Thermal limit, based on the approximate current-carrying capacity listed in Table A.4

e. Transmission-line efficiency at full load

SOLUTION

a. The total series impedance and shunt admittance values are

$$Z = zl = (0.032 + j0.35)(200) = 6.4 + j70 = 70.29\underline{/84.78^\circ} \quad \Omega$$

$$Y = yl = (j4.2 \times 10^{-6})(200) = 8.4 \times 10^{-4}\underline{/90^\circ} \quad \text{S}$$

From (5.1.15)–(5.1.17),

$$A = D = 1 + (8.4 \times 10^{-4}\underline{/90^\circ})(70.29\underline{/84.78^\circ})\left(\tfrac{1}{2}\right)$$

$$= 1 + 0.02952\underline{/174.78^\circ}$$

$$= 0.9706 + j0.00269 = 0.9706\underline{/0.159^\circ} \quad \text{per unit}$$

$$B = Z = 70.29\underline{/84.78^\circ} \quad \Omega$$

$$C = (8.4 \times 10^{-4}\underline{/90^\circ})(1 + 0.01476\underline{/174.78^\circ})$$

$$= (8.4 \times 10^{-4}\underline{/90^\circ})(0.9853 + j0.00134)$$

$$= 8.277 \times 10^{-4}\underline{/90.08^\circ} \quad \text{S}$$

b. The receiving-end voltage and current quantities are

$$\text{V}_\text{R} = (0.95)(345) = 327.8 \quad \text{kV}_\text{LL}$$

$$V_\text{R} = \frac{327.8}{\sqrt{3}}\underline{/0^\circ} = 189.2\underline{/0^\circ} \quad \text{kV}_\text{LN}$$

$$I_\text{R} = \frac{700\underline{/\cos^{-1} 0.99}}{(\sqrt{3})(0.95 \times 345)(0.99)} = 1.246\underline{/8.11^\circ} \quad \text{kA}$$

From (5.1.1) and (5.1.2), the sending-end quantities are

$$V_\text{S} = (0.9706\underline{/0.159^\circ})(189.2\underline{/0^\circ}) + (70.29\underline{/84.78^\circ})(1.246\underline{/8.11^\circ})$$

$$= 183.6\underline{/0.159^\circ} + 87.55\underline{/92.89^\circ}$$

$$= 179.2 + j87.95 = 199.6\underline{/26.14^\circ} \quad \text{kV}_\text{LN}$$

$$\text{V}_\text{S} = 199.6\sqrt{3} = 345.8 \text{ kV}_\text{LL} \approx 1.00 \quad \text{per unit}$$

$$I_\text{S} = (8.277 \times 10^{-4}\underline{/90.08^\circ})(189.2\underline{/0^\circ}) + (0.9706\underline{/0.159^\circ})(1.246\underline{/8.11^\circ})$$

$$= 0.1566\underline{/90.08^\circ} + 1.209\underline{/8.27^\circ}$$

$$= 1.196 + j0.331 = 1.241\underline{/15.5^\circ} \quad \text{kA}$$

and the real power delivered to the sending end is

$$\text{P}_\text{S} = (\sqrt{3})(345.8)(1.241)\cos(26.14^\circ - 15.5^\circ)$$

$$= 730.5 \quad \text{MW}$$

c. From (5.1.19), the no-load receiving-end voltage is

$$\text{V}_\text{RNL} = \frac{\text{V}_\text{S}}{\text{A}} = \frac{345.8}{0.9706} = 356.3 \quad \text{kV}_\text{LL}$$

and, from (5.1.18),

$$\text{percent VR} = \frac{356.3 - 327.8}{327.8} \times 100 = 8.7\%$$

d. From Table A.4, the approximate current-carrying capacity of two 795,000-cmil 26/2 ACSR conductors is $2 \times 0.9 = 1.8$ kA.

e. The full-load line losses are $\text{P}_\text{S} - \text{P}_\text{R} = 730.5 - 700 = 30.5$ MW and the full-load transmission efficiency is

$$\text{percent EFF} = \frac{\text{P}_\text{R}}{\text{P}_\text{S}} \times 100 = \frac{700}{730.5} \times 100 = 95.8\%$$

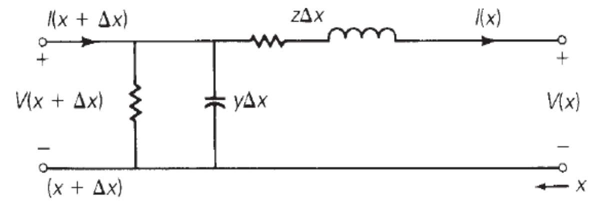
Since $V_S = 1.00$ per unit, the full-load receiving-end voltage of 0.95 per unit corresponds to $V_R/V_S = 0.95$, considered in practice to be about the lowest operating voltage possible without encountering operating problems. Thus, for this 345-kV 200-km uncompensated line, voltage drop limits the full-load current to 1.246 kA at 0.99 p.f. leading, well below the thermal limit of 1.8 kA. ■

5.2. Transmittion Line Differential Equations: (Exact ABCD parameters)

$$z = R + j\omega L \quad \Omega/\text{m}$$

$$y = G + j\omega C \quad \text{S/m}$$

FIGURE 5.6 Transmission-line section of length Δx



By KVL:

$$V(x+\Delta x) = V(x) + (z\Delta x)I(x) \quad \text{volts} \Rightarrow \underbrace{\frac{V(x+\Delta x) - V(x)}{\Delta x} = zI(x)}_{\lim\limits_{\Delta x \to 0}} \Rightarrow \frac{dV(x)}{dx} = zI(x)$$

By KCL:

$$I(x+\Delta x) = I(x) + (y\Delta x)V(x+\Delta x) \quad \text{A} \Rightarrow \underbrace{\frac{I(x+\Delta x) - I(x)}{\Delta x} = yV(x)}_{\lim\limits_{\Delta x \to 0}} \Rightarrow \frac{dI(x)}{dx} = yV(x)$$

$$\frac{d^2V(x)}{dx^2} = z\frac{dI(x)}{dx} = zyV(x)$$

or

$$\frac{d^2V(x)}{dx^2} - zyV(x) = 0$$

The solution of the diffirential equation is:

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \quad \text{volts}$$, where $\gamma = \sqrt{zy}$ m^{-1} $= \alpha + j\beta$ (propagation constant)

$$\frac{dV(x)}{dx} = \gamma A_1 e^{\gamma x} - \gamma A_2 e^{-\gamma x} = zI(x)$$

Solving for $I(x)$:

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{z/\gamma}$$

$$I(x)=\frac{A_1e^{\gamma x}-A_2e^{-\gamma x}}{Z_c}$$, where $Z_c=\sqrt{\frac{Z}{Y}}$ Ω (called characteristic impedance)

* Finding A_1 & A_2 :

at $x=0$ (Receiving-end) :

$$\begin{cases} V_R=V(0) \\ I_R=I(0) \end{cases}$$

$$\left.\begin{aligned} V_R&=A_1+A_2 \\ I_R&=\frac{A_1-A_2}{Z_c} \end{aligned}\right\}$$ soling for A_1 & A_2 :

$$A_1=\frac{V_R+Z_cI_R}{2}$$

$$A_2=\frac{V_R-Z_cI_R}{2}$$

$$V(x)=\left(\frac{V_R+Z_cI_R}{2}\right)e^{\gamma x}+\left(\frac{V_R-Z_cI_R}{2}\right)e^{-\gamma x}=\left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right)V_R+Z_c\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)I_R=\cosh(\gamma x)V_R+Z_c\sinh(\gamma x)I_R$$

$$I(x)=\left(\frac{V_R+Z_cI_R}{2Z_c}\right)e^{\gamma x}-\left(\frac{V_R-Z_cI_R}{2Z_c}\right)e^{-\gamma x}=\frac{1}{Z_c}\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)V_R+\left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right)I_R=\frac{1}{Z_c}\sinh(\gamma x)V_R+\cosh(\gamma x)I_R$$

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix}=\left[\begin{array}{c|c} A(x) & B(x) \\ \hline C(x) & D(x) \end{array}\right]\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

at $x=l$ (the sending-end):

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix}=\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

where

$$A=D=\cosh(\gamma l) \quad \text{per unit}$$

$$B=Z_c\sinh(\gamma l) \quad \Omega$$

$$C=\frac{1}{Z_c}\sinh(\gamma l) \quad \text{S}$$

Parameter	$A=D$	B	C
Units	per Unit	Ω	S
Short line (less than 80 km)	1	Z	0
Medium line nominal π circuit (80 to 250 km)	$1+\frac{YZ}{2}$	Z	$Y\left(1+\frac{YZ}{4}\right)$
Long line equivalent π circuit (more than 250 km)	$\cosh(\gamma\ell)=1+\frac{Y'Z'}{2}$	$Z_c\sinh(\gamma\ell)=Z'$	$(1/Z_c)\sinh(\gamma\ell)$ $=Y'\left(1+\frac{Y'Z'}{4}\right)$
Lossless line ($R=G=0$)	$\cos(\beta\ell)$	$jZ_c\sin(\beta\ell)$	$\frac{j\sin(\beta\ell)}{Z_c}$

EXAMPLE 5.2 Exact *ABCD* parameters: long line

A three-phase 765-kV, 60-Hz, 300-km, completely transposed line has the following positive-sequence impedance and admittance:

$$z = 0.0165 + j0.3306 = 0.3310\underline{/87.14^\circ} \quad \Omega/\text{km}$$

$$y = j4.674 \times 10^{-6} \quad \text{S/km}$$

Assuming positive-sequence operation, calculate the exact *ABCD* parameters of the line. Compare the exact B parameter with that of the nominal π circuit.

SOLUTION From (5.2.12) and (5.2.16):

$$Z_c = \sqrt{\frac{0.3310\underline{/87.14^\circ}}{4.674 \times 10^{-6}\underline{/90^\circ}}} = \sqrt{7.082 \times 10^4\underline{/-2.86^\circ}}$$

$$= 266.1\underline{/-1.43^\circ} \quad \Omega$$

and

$$\gamma l = \sqrt{(0.3310\underline{/87.14^\circ})(4.674 \times 10^{-6}\underline{/90^\circ})} \times (300)$$

$$= \sqrt{1.547 \times 10^{-6}\underline{/177.14^\circ}} \times (300)$$

$$= 0.3731\underline{/88.57^\circ} = 0.00931 + j0.3730 \quad \text{per unit}$$

From (5.2.38),

$$e^{\gamma l} = e^{0.00931}e^{+j0.3730} = 1.0094\underline{/0.3730} \quad \text{radians}$$

$$= 0.9400 + j0.3678$$

and

$$e^{-\gamma l} = e^{-0.00931}e^{-j0.3730} = 0.9907\underline{/-0.3730} \quad \text{radians}$$

$$= 0.9226 - j0.3610$$

Then, from (5.2.39) and (5.2.40),

$$\cosh(\gamma l) = \frac{(0.9400 + j0.3678) + (0.9226 - j0.3610)}{2}$$

$$= 0.9313 + j0.0034 = 0.9313\underline{/0.209^\circ}$$

$$\sinh(\gamma l) = \frac{(0.9400 + j0.3678) - (0.9226 - j0.3610)}{2}$$

$$= 0.0087 + j0.3644 = 0.3645\underline{/88.63^\circ}$$

Finally, from (5.2.34)–(5.2.36),

$$A = D = \cosh(\gamma l) = 0.9313\underline{/0.209^\circ} \quad \text{per unit}$$

$$B = (266.1\underline{/-1.43^\circ})(0.3645\underline{/88.63^\circ}) = 97.0\underline{/87.2^\circ} \quad \Omega$$

$$C = \frac{0.3645\underline{/88.63^\circ}}{266.1\underline{/-1.43^\circ}} = 1.37 \times 10^{-3}\underline{/90.06^\circ} \quad \text{S}$$

Using (5.1.16), the B parameter for the nominal π circuit is

$$B_{\text{nominal }\pi} = Z = (0.3310\underline{/87.14^\circ})(300) = 99.3\underline{/87.14^\circ} \quad \Omega$$

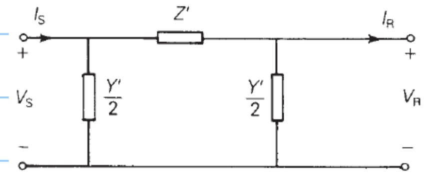
which is 2% larger than the exact value. ■

5.3 Equivalent π Circuit: (Exact ABCD parameters)

$$A = D = 1 + \frac{Y'Z'}{2} \quad \text{per unit}$$

$$B = Z' \quad \Omega$$

$$C = Y'\left(1 + \frac{Y'Z'}{4}\right) \quad \text{S}$$



$$Z' = Z_c \sinh(\gamma\ell) = ZF_1 = Z\frac{\sinh(\gamma\ell)}{\gamma\ell}$$

$$\frac{Y'}{2} = \frac{\tanh(\gamma\ell/2)}{Z_c} = \frac{Y}{2}F_2 = \frac{Y}{2}\frac{\tanh(\gamma\ell/2)}{(\gamma\ell/2)}$$

(Equivalent π circuit)

$$Z' = \underbrace{Z_c}\sinh(\gamma l) = \left(\sqrt{\frac{z}{y}}\right)\sinh(\gamma l) = zl\left[\sqrt{\frac{z}{y}}\,\frac{\sinh(\gamma l)}{zl}\right] = \underbrace{zl}\overbrace{\left[\frac{\sinh(\gamma l)}{\sqrt{zy}\,l}\right]} = ZF_1 \quad \Omega$$

$$1 + \frac{Y'Z'}{2} = \cosh(\gamma l) \Rightarrow \frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z'} = \frac{\cosh(\gamma l) - 1}{Z_c \sinh(\gamma l)} = \frac{\tanh(\gamma l/2)}{Z_c} = \frac{\tanh(\gamma l/2)}{\sqrt{\frac{z}{y}}}$$

$$\frac{Y'}{2} = \frac{yl}{2}\left[\frac{\tanh(\gamma l/2)}{\sqrt{\frac{z}{y}}\,\frac{yl}{2}}\right] = \overbrace{\frac{yl}{2}}\underbrace{\left[\frac{\tanh(\gamma l/2)}{\sqrt{zy}\,l/2}\right]} = \frac{Y}{2}F_2 \quad \text{S}$$

, F_1 & F_2 are correction factors to convert nominal π circuit into equivalent π circuit.

EXAMPLE 5.3 Equivalent π circuit: long line

Compare the equivalent and nominal π circuits for the line in Example 5.2.

SOLUTION For the nominal π circuit,

$$Z = zl = (0.3310\underline{/87.14^\circ})(300) = 99.3\underline{/87.14^\circ} \quad \Omega$$

$$\frac{Y}{2} = \frac{yl}{2} = \left(\frac{j4.674 \times 10^{-6}}{2}\right)(300) = 7.011 \times 10^{-4}\underline{/90^\circ} \quad \text{S}$$

From (5.3.6) and (5.3.10), the correction factors are

$$F_1 = \frac{0.3645\underline{/88.63^\circ}}{0.3731\underline{/88.57^\circ}} = 0.9769\underline{/0.06^\circ} \quad \text{per unit}$$

$$F_2 = \frac{\tanh(\gamma l/2)}{\gamma l/2} = \frac{\cosh(\gamma l) - 1}{(\gamma l/2)\sinh(\gamma l)}$$

$$= \frac{0.9313 + j0.0034 - 1}{\left(\frac{0.3731}{2}\underline{/88.57^\circ}\right)(0.3645\underline{/88.63^\circ})}$$

$$= \frac{-0.0687 + j0.0034}{0.06800\underline{/177.20^\circ}}$$

$$= \frac{0.06878\underline{/177.17^\circ}}{0.06800\underline{/177.20^\circ}} = 1.012\underline{/-0.03^\circ} \quad \text{per unit}$$

Then, from (5.3.5) and (5.3.9), for the equivalent π circuit,

$$Z' = (99.3\underline{/87.14^\circ})(0.9769\underline{/0.06^\circ}) = 97.0\underline{/87.2^\circ} \quad \Omega$$

$$\frac{Y'}{2} = (7.011 \times 10^{-4}\underline{/90^\circ})(1.012\underline{/-0.03^\circ}) = 7.095 \times 10^{-4}\underline{/89.97^\circ} \quad \text{S}$$

$$= 3.7 \times 10^{-7} + j7.095 \times 10^{-4} \quad \text{S}$$

Comparing these nominal and equivalent π circuit values, Z' is about 2% smaller than Z, and $Y'/2$ is about 1% larger than $Y/2$. Although the circuit values are approximately the same for this line, the equivalent π circuit should be used for accurate calculations involving long lines. Note the small shunt conductance, $G' = 3.7 \times 10^{-7}$ S, introduced in the equivalent π circuit. G' is often neglected. ■

5.4 Lossless Lines: (R = G = 0)

a) Surge Impedance: (characteristic impedance for lossess lines)

$$\left.\begin{aligned} z &= j\omega \mathrm{L} \quad \Omega/\mathrm{m} \\ y &= j\omega \mathrm{C} \quad \mathrm{S}/\mathrm{m} \end{aligned}\right\} \quad Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega \mathrm{L}}{j\omega \mathrm{C}}} = \sqrt{\frac{\mathrm{L}}{\mathrm{C}}} \quad \Omega \quad \text{(real)}$$

and

$$\gamma = \sqrt{zy} = \sqrt{(j\omega \mathrm{L})(j\omega \mathrm{C})} = j\omega\sqrt{\mathrm{LC}} = j\beta \quad \mathrm{m}^{-1} \quad \text{(imaginary)}$$

b) ABCD Parameters:

$$\mathrm{A}(x) = \mathrm{D}(x) = \cosh(\gamma x) = \cosh(j\beta x)$$

$$= \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x) \quad \text{per unit} \quad \text{(real)}$$

$$\sinh(\gamma x) = \sinh(j\beta x) = \frac{e^{j\beta x} - e^{-j\beta x}}{2} = j\sin(\beta x) \quad \text{per unit}$$

$$B(x) = \mathrm{Z}_c \sinh(\gamma x) = j\mathrm{Z}_c \sin(\beta x) = j\sqrt{\frac{\mathrm{L}}{\mathrm{C}}}\sin(\beta x) \quad \Omega \quad \text{(imaginary)}$$

$$C(x) = \frac{\sinh(\gamma x)}{\mathrm{Z}_c} = \frac{j\sin(\beta x)}{\sqrt{\frac{\mathrm{L}}{\mathrm{C}}}} \quad \mathrm{S} \quad \text{(imaginary)}$$

$\mathrm{A}(x)$ and $\mathrm{D}(x)$ are pure real; $B(x)$ and $C(x)$ are pure imaginary.

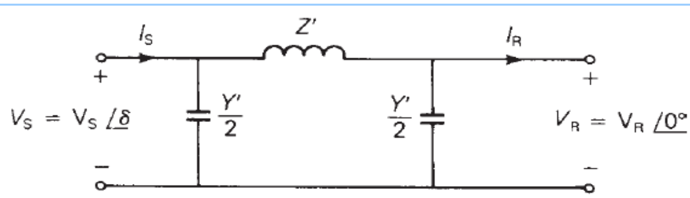
c) Equivalent π Circuit:

$$Z' = j\mathrm{Z}_c \sin(\beta l) = \underbrace{(j\omega \mathrm{L} l)}_{X_L}\left(\frac{\sin(\beta l)}{\beta l}\right) = j\mathrm{X}' \quad \Omega \quad \text{(imaginary)}$$

$$\frac{Y'}{2} = \frac{Y}{2}\frac{\tanh(j\beta l/2)}{j\beta l/2} = \frac{Y}{2}\frac{\sinh(j\beta l/2)}{(j\beta l/2)\cosh(j\beta l/2)} = \left(\frac{j\omega Cl}{2}\right)\frac{j\sin(\beta l/2)}{(j\beta l/2)\cos(\beta l/2)} = \left(\frac{j\omega Cl}{2}\right)\frac{\tan(\beta l/2)}{\beta l/2} = \left(\frac{j\omega C'l}{2}\right) \quad \mathrm{S}$$

(imaginary)

Equivalent π circuit for a lossless line ($\beta\ell$ less than π)



$$Z' = (j\omega \mathrm{L}\ell)\left(\frac{\sin\beta\ell}{\beta\ell}\right) = j\mathrm{X}' \quad \Omega$$

$$\frac{Y'}{2} = \left(\frac{j\omega \mathrm{C}\ell}{2}\right)\frac{\tan(\beta\ell/2)}{(\beta\ell/2)} = \frac{j\omega \mathrm{C}'\ell}{2} \quad \mathrm{S}$$

d) Wavelength: (λ)

$$V(x) = A(x)V_R + B(x)I_R$$
$$= \cos(\beta x)V_R + jZ_c \sin(\beta x)I_R$$

و

$$I(x) = C(x)V_R + D(x)I_R$$
$$= \frac{j\sin(\beta x)}{Z_c}V_R + \cos(\beta x)I_R$$

wavelength $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}} \quad \text{m}$ or $f\lambda = \frac{1}{\sqrt{LC}}$

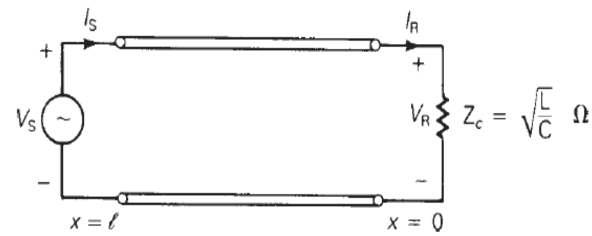
$$\lambda \approx \frac{3\times 10^8}{60} = 5\times 10^6 \text{ m} = 5000 \text{ km} = 3100 \text{ mi}$$

e) Surge Impedance Loading (SIL):

$$V(x) = \cos(\beta x)V_R + jZ_c \sin(\beta x)I_R$$
$$= \cos(\beta x)V_R + jZ_c \sin(\beta x)\left(\frac{V_R}{Z_c}\right)$$
$$= (\cos\beta x + j\sin\beta x)V_R$$
$$= e^{j\beta x}V_R \quad \text{volts}$$
$$|V(x)| = |V_R| \quad \text{volts}$$

FIGURE 5.9

Lossless line terminated by its surge impedance



V_{rated} (kV)	$Z_c = \sqrt{L/C}$ (Ω)	SIL$= V^2_{rated}/Z_c$ (MW)
69	366–400	12–13
138	366–405	47–52
230	365–395	134–145
345	280–366	325–425
500	233–294	850–1075
765	254–266	2200–2300

$$I(x) = \frac{j\sin(\beta x)}{Z_c}V_R + (\cos\beta x)\frac{V_R}{Z_c}$$
$$= (\cos\beta x + j\sin\beta x)\frac{V_R}{Z_c}$$
$$= (e^{j\beta x})\frac{V_R}{Z_c} \quad \text{A}$$

و

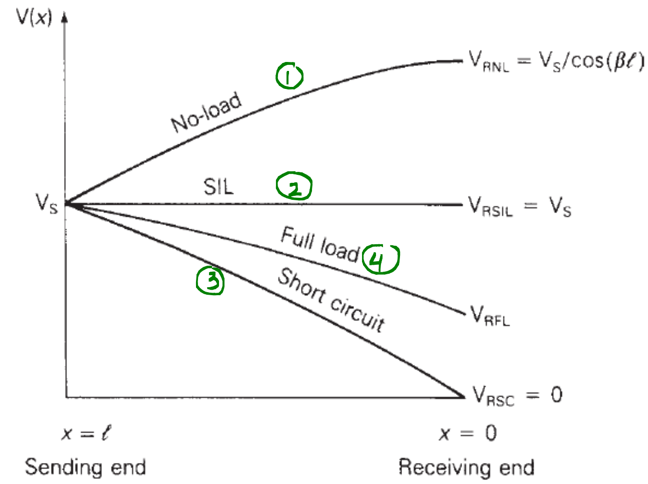
$$S(x) = P(x) + jQ(x) = V(x)I^*(x)$$
$$= (e^{j\beta x}V_R)\left(\frac{e^{j\beta x}V_R}{Z_c}\right)^*$$
$$= \frac{|V_R|^2}{Z_c}$$

P is constant & Q = 0

$$\text{SIL} = \frac{V^2_{rated}}{Z_c}$$

f) Voltage Profiles :

1. At no-load, $I_{RNL} = 0$ and (5.4.13) yields
$$V_{NL}(x) = (\cos \beta x) V_{RNL} \quad (5.4.22)$$
The no-load voltage increases from $V_S = (\cos \beta l) V_{RNL}$ at the sending end to V_{RNL} at the receiving end (where $x = 0$).
2. From (5.4.18), the voltage profile at SIL is flat.
3. For a short circuit at the load, $V_{RSC} = 0$ and (5.4.13) yields
$$V_{SC}(x) = (Z_c \sin \beta x) I_{RSC} \quad (5.4.23)$$
The voltage decreases from $V_S = (\sin \beta l)(Z_c I_{RSC})$ at the sending end to $V_{RSC} = 0$ at the receiving end.
4. The full-load voltage profile, which depends on the specification of full-load current, lies above the short-circuit voltage profile.



g) Steady-State Stability Limit:

From the equivalent π circuit:

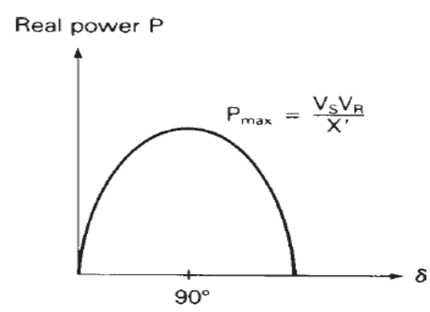
$$I_R = \frac{V_S - V_R}{Z'} - \frac{Y'}{2} V_R$$
$$= \frac{V_S e^{j\delta} - V_R}{jX'} - \frac{j\omega C' l}{2} V_R$$

$$S_R = V_R I_R^* = V_R \left(\frac{V_S e^{j\delta} - V_R}{jX'} \right)^* + \frac{j\omega C' l}{2} V_R^2$$
$$= V_R \left(\frac{V_S e^{-j\delta} - V_R}{-jX'} \right) + \frac{j\omega C l}{2} V_R^2$$
$$= \frac{jV_R V_S \cos\delta + \boxed{V_R V_S \sin\delta} - jV_R^2}{X'} + \frac{j\omega C l}{2} V_R^2$$

$$P = P_S = P_R = \mathrm{Re}(S_R) = \frac{V_R V_S}{X'} \sin\delta \quad W \Rightarrow P_{max} = \frac{V_S V_R}{X'} \quad W, \ \delta = 90°$$

FIGURE 5.11

Real power delivered by a lossless line versus voltage angle across the line



$$P = \frac{V_S V_R \sin\delta}{Z_c \sin\beta l} = \left(\frac{V_S V_R}{Z_c}\right) \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

Expressing V_S and V_R in per-unit of rated line voltage,

$$P = \left(\frac{V_S}{V_{rated}}\right)\left(\frac{V_R}{V_{rated}}\right)\left(\frac{V_{rated}^2}{Z_c}\right) \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

$$= V_{S.p.u.} V_{R.p.u.} (SIL) \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)} \quad W$$

And for $\delta = 90°$, the theoretical steady-state stability limit is

$$P_{max} = \frac{V_{S.p.u.} V_{R.p.u.} (SIL)}{\sin\left(\frac{2\pi l}{\lambda}\right)} \quad W$$, $V \uparrow \Rightarrow P_{max} \uparrow$, $l \uparrow \Rightarrow P_{max} \downarrow$

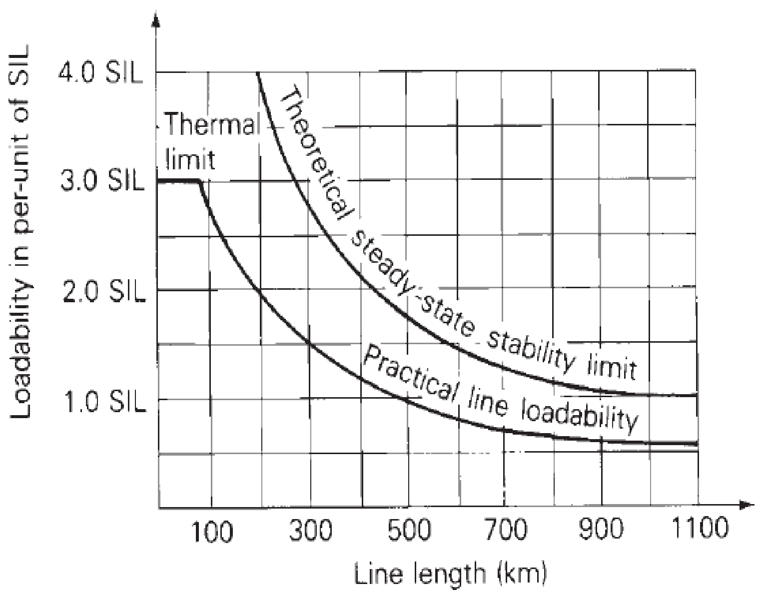


FIGURE 5.12 Transmission-line loadability curve for 60-Hz overhead lines—no series or shunt compensation

EXAMPLE 5.4 Theoretical steady-state stability limit: long line

Neglecting line losses, find the theoretical steady-state stability limit for the 300-km line in Example 5.2. Assume a 266.1-Ω surge impedance, a 5000-km wavelength, and $V_S = V_R = 765$ kV.

SOLUTION From (5.4.21),

$$SIL = \frac{(765)^2}{266.1} = 2199 \quad MW$$

From (5.4.30) with $l = 300$ km and $\lambda = 5000$ km,

$$P_{max} = \frac{(1)(1)(2199)}{\sin\left(\frac{2\pi \times 300}{5000}\right)} = (2.716)(2199) = 5974 \quad MW$$

show PW

5.5 Maximum Power Flow: (Lossy lines)

$$A = \cosh(\gamma l) = \mathrm{A}\angle\theta_{\mathrm{A}}$$

$$B = Z' = \mathrm{Z}'\angle\theta_{\mathrm{Z}}$$

$$V_{\mathrm{S}} = \mathrm{V}_{\mathrm{S}}\angle\delta \qquad V_{\mathrm{R}} = \mathrm{V}_{\mathrm{R}}\angle 0°$$

$$I_{\mathrm{R}} = \frac{V_{\mathrm{S}} - AV_{\mathrm{R}}}{B} = \frac{\mathrm{V}_{\mathrm{S}}e^{j\delta} - \mathrm{AV}_{\mathrm{R}}e^{j\theta_{\mathrm{A}}}}{\mathrm{Z}'e^{j\theta_{\mathrm{Z}}}}$$

The complex power delivered to the receiving end is

$$S_{\mathrm{R}} = \mathrm{P}_{\mathrm{R}} + j\mathrm{Q}_{\mathrm{R}} = V_{\mathrm{R}}I_{\mathrm{R}}^{*} = \mathrm{V}_{\mathrm{R}}\left[\frac{\mathrm{V}_{\mathrm{S}}e^{j(\delta-\theta_{\mathrm{Z}})} - \mathrm{AV}_{\mathrm{R}}e^{j(\theta_{\mathrm{A}}-\theta_{\mathrm{Z}})}}{\mathrm{Z}'}\right]^{*}$$

$$= \frac{\mathrm{V}_{\mathrm{R}}\mathrm{V}_{\mathrm{S}}}{\mathrm{Z}'}e^{j(\theta_{\mathrm{Z}}-\delta)} - \frac{\mathrm{AV}_{\mathrm{R}}^{2}}{\mathrm{Z}'}e^{j(\theta_{\mathrm{Z}}-\theta_{\mathrm{A}})}$$

The real and reactive power delivered to the receiving end are thus

$$\mathrm{P}_{\mathrm{R}} = \mathrm{Re}(S_{\mathrm{R}}) = \frac{\mathrm{V}_{\mathrm{R}}\mathrm{V}_{\mathrm{S}}}{\mathrm{Z}'}\cos(\theta_{\mathrm{Z}}-\delta) - \frac{\mathrm{AV}_{\mathrm{R}}^{2}}{\mathrm{Z}'}\cos(\theta_{\mathrm{Z}}-\theta_{\mathrm{A}})$$

$$\mathrm{Q}_{\mathrm{R}} = \mathrm{Im}(S_{\mathrm{R}}) = \frac{V_{\mathrm{R}}V_{\mathrm{S}}}{Z'}\sin(\theta_{\mathrm{Z}}-\delta) - \frac{\mathrm{AV}_{\mathrm{R}}^{2}}{\mathrm{Z}'}\sin(\theta_{\mathrm{Z}}-\theta_{\mathrm{A}})$$

$$\mathrm{P}_{\mathrm{Rmax}} = \frac{\mathrm{V}_{\mathrm{R}}\mathrm{V}_{\mathrm{S}}}{\mathrm{Z}'} - \frac{\mathrm{AV}_{\mathrm{R}}^{2}}{\mathrm{Z}'}\cos(\theta_{\mathrm{Z}}-\theta_{\mathrm{A}})$$, when $\delta = \theta_Z$

Read Ex 5.5

5.6 Line Loadability:

Practical line loadability:

1. $V_R/V_S \geqslant 0.95$
2. $\delta \leqslant 35°$
3. Limited by thermal rating. ($l < 80$km)

EXAMPLE 5.6 Practical line loadability and percent voltage regulation: long line

The 300-km uncompensated line in Example 5.2 has four 1,272,000-cmil 54/3 ACSR conductors per bundle. The sending-end voltage is held constant at 1.0 per-unit of rated line voltage. Determine the following:

a. The practical line loadability. (Assume an approximate receiving-end voltage $V_R = 0.95$ per unit and $\delta = 35°$ maximum angle across the line.)

b. The full-load current at 0.986 p.f. leading based on the above practical line loadability

c. The exact receiving-end voltage for the full-load current found in part (b)

d. Percent voltage regulation for the above full-load current

e. Thermal limit of the line, based on the approximate current-carrying capacity given in Table A.4

SOLUTION

a. From (5.5.3), with $V_S = 765$, $V_R = 0.95 \times 765$ kV, and $\delta = 35°$, using the values of Z', θ_Z, A, and θ_A from Example 5.5,

$$P_R = \frac{(765)(0.95 \times 765)}{97.0}\cos(87.2° - 35°) - \frac{(0.9313)(0.95 \times 765)^2}{97.0}\cos(87.2° - 0.209°)$$
$$= 3513 - 266 = 3247 \quad \text{MW}$$

$P_R = 3247$ MW is the practical line loadability, provided the thermal and voltage-drop limits are not exceeded. Alternatively, from Figure 5.12 for a 300-km line, the practical line loadability is (1.49)SIL = (1.49)(2199) = 3277 MW, about the same as the above result.

b. For the above loading at 0.986 p.f. leading and at 0.95×765 kV, the full-load receiving-end current is

$$I_{RFL} = \frac{P}{\sqrt{3}V_R(\text{p.f.})} = \frac{3247}{(\sqrt{3})(0.95 \times 765)(0.986)} = 2.616 \quad \text{kA}$$

c. From (5.1.1) with $I_{RFL} = 2.616\underline{/\cos^{-1} 0.986} = 2.616\underline{/9.599°}$ kA, using the A and B parameters from Example 5.2,

$$V_S = AV_{RFL} + BI_{RFL}$$

$$\frac{765}{\sqrt{3}}\underline{/\delta} = (0.9313\underline{/0.209°})(V_{RFL}\underline{/0°}) + (97.0\underline{/87.2°})(2.616\underline{/9.599°})$$

$$441.7\underline{/\delta} = (0.9313V_{RFL} - 30.04) + j(0.0034V_{RFL} + 251.97)$$

Taking the squared magnitude of the above equation,

$$(441.7)^2 = 0.8673V_{RFL}^2 - 54.24V_{RFL} + 64{,}391$$

Solving,

$$V_{RFL} = 420.7 \quad \text{kV}_{LN}$$
$$= 420.7\sqrt{3} = 728.7 \quad \text{kV}_{LL} = 0.953 \quad \text{per unit}$$

d. From (5.1.19), the receiving-end no-load voltage is

$$V_{RNL} = \frac{V_S}{A} = \frac{765}{0.9313} = 821.4 \quad kV_{LL}$$

And from (5.1.18),

$$\text{percent VR} = \frac{821.4 - 728.7}{728.7} \times 100 = 12.72\%$$

e. From Table A.4, the approximate current-carrying capacity of four 1,272,000-cmil 54/3 ACSR conductors is $4 \times 1.2 = 4.8$ kA.

Since the voltages $V_S = 1.0$ and $V_{RFL} = 0.953$ per unit satisfy the voltage-drop limit $V_R/V_S \geq 0.95$, the factor that limits line loadability is steady-state stability for this 300-km uncompensated line. The full-load current of 2.616 kA corresponding to loadability is also well below the thermal limit of 4.8 kA. The 12.7% voltage regulation is too high because the no-load voltage is too high. Compensation techniques to reduce no-load voltages are discussed in Section 5.7. ■

Read 5.7 & 5.8

5.7 Reactive Compensation Techniques:

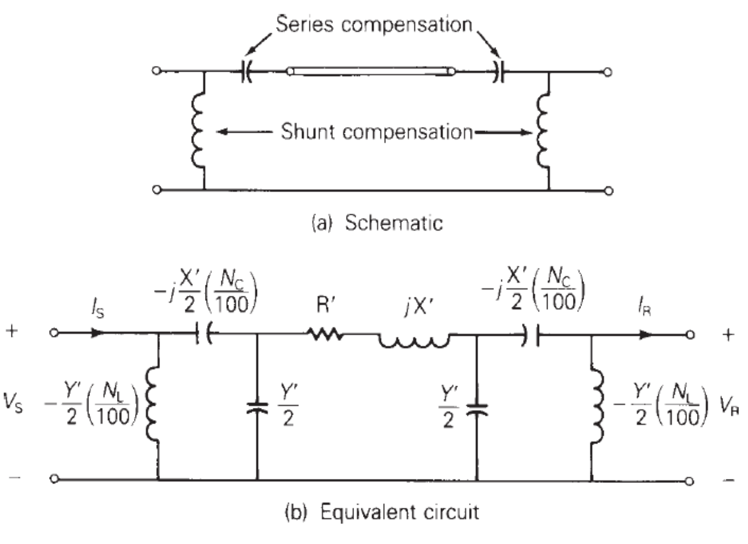
* Shunt reactors: Connected to EHV lines to absorb Q and reduce voltage to the rated value during light loading.

* Shunt capacitors: Supply Q to increase the voltage during heavy loading. (SVC)

* Series capacitors: Increase line loadability and decrease line voltage drop.

FIGURE 5.16

Compensated transmission-line section



N_L : percentage of shunt admittance

N_c : " " sereies impedance

EXAMPLE 5.9 **Shunt reactive compensation to improve transmission-line voltage regulation**

Identical shunt reactors (inductors) are connected from each phase conductor to neutral at both ends of the 300-km line in Example 5.2 during light load conditions, providing 75% compensation. The reactors are removed during heavy load conditions. Full load is 1.90 kA at unity p.f. and at 730 kV. Assuming that the sending-end voltage is constant, determine the following:

a. Percent voltage regulation of the uncompensated line

b. The equivalent shunt admittance and series impedance of the compensated line

c. Percent voltage regulation of the compensated line

SOLUTION

a. From (5.1.1) with $I_{RFL} = 1.9\underline{/0^\circ}$ kA, using the A and B parameters from Example 5.2,

$$\begin{aligned} V_S &= AV_{RFL} + BI_{RFL} \\ &= (0.9313\underline{/0.209^\circ})\left(\frac{730}{\sqrt{3}}\underline{/0^\circ}\right) + (97.0\underline{/87.2^\circ})(1.9\underline{/0^\circ}) \\ &= 392.5\underline{/0.209^\circ} + 184.3\underline{/87.2^\circ} \\ &= 401.5 + j185.5 \\ &= 442.3\underline{/24.8^\circ} \quad \text{kV}_{LN} \end{aligned}$$

$$\text{V}_S = 442.3\sqrt{3} = 766.0 \quad \text{kV}_{LL}$$

The no-load receiving-end voltage is, from (5.1.19),

$$\text{V}_{RNL} = \frac{766.0}{0.9313} = 822.6 \quad \text{kV}_{LL}$$

and the percent voltage regulation for the uncompensated line is, from (5.1.18),

$$\text{percent VR} = \frac{822.6 - 730}{730} \times 100 = 12.68\%$$

b. From Example 5.3, the shunt admittance of the equivalent π circuit without compensation is

$$\begin{aligned} Y' &= 2(3.7 \times 10^{-7} + j7.094 \times 10^{-4}) \\ &= 7.4 \times 10^{-7} + j14.188 \times 10^{-4} \quad \text{S} \end{aligned}$$

With 75% shunt compensation, the equivalent shunt admittance is

$$\begin{aligned} Y_{eq} &= 7.4 \times 10^{-7} + j14.188 \times 10^{-4}\left(1 - \tfrac{75}{100}\right) \\ &= 3.547 \times 10^{-4}\underline{/89.88^\circ} \quad \text{S} \end{aligned}$$

Since there is no series compensation, the equivalent series impedance is the same as without compensation:

$$Z_{eq} = Z' = 97.0\underline{/87.2^\circ} \quad \Omega$$

c. The equivalent A parameter for the compensated line is

$$\begin{aligned} A_{eq} &= 1 + \frac{Y_{eq}Z_{eq}}{2} \\ &= 1 + \frac{(3.547 \times 10^{-4}\underline{/89.88^\circ})(97.0\underline{/87.2^\circ})}{2} \\ &= 1 + 0.0172\underline{/177.1^\circ} \end{aligned}$$

$$= 1 + 0.0172\underline{/177.1^\circ}$$

$$= 0.9828\underline{/0.05^\circ} \quad \text{per unit}$$

Then, from (5.1.19),

$$V_{RNL} = \frac{766}{0.9828} = 779.4 \quad kV_{LL}$$

Since the shunt reactors are removed during heavy load conditions, $V_{RFL} = 730$ kV is the same as without compensation. Therefore

$$\text{percent VR} = \frac{779.4 - 730}{730} \times 100 = 6.77\%$$

The use of shunt reactors at light loads improves the voltage regulation from 12.68% to 6.77% for this line. ■

show PW